**SECOND TERM E-LEARNING NOTE**

**SUBJECT: MATHEMATICS CLASS: SS 3**

**SCHEME OF WORK**

**WEEK TOPIC**

1. **Calculation on interest on bonds and debentures using logarithm table and problems on taxes and value added tax.**
2. **Coordinate Geometry of straight line: Cartesian coordinate graphs, distance between two points, midpoint of the line joining two points.**
3. **Coordinate Geometry of straight lines: Gradient and Intercepts of a line, angle between two intersecting straight lines and application.**
4. **Differentiation of algebraic functions: meaning of differentiation, differentiation from first principle and standard derivatives of some basic functions.**
5. **Differentiation of algebraic functions: Basic rules of differentiation such as sum and difference, product rule, quotient rule and maximal and minimum application.**
6. **Integration and evaluation of simple algebraic functions: Definition, method of integration: substitution, partial fraction and integration by parts, area under the curve and use of Simpson’s rule.**
7. **– 12. Revision and mock examination.**

**REFERENCE TEXT**

* **New General Mathematics for SS book 3 by J.B Channon**
* **Essential Mathematics for SS book 3**
* **Mathematics Exam Focus**
* **Waec and Jamb past Questions**

**WEEK ONE**

* Calculation on interest on bonds and debentures using logarithm table
* Problems on taxes and value added tax.

**WEEK TWO**

* Coordinate Geometry of straight line: Cartesian coordinate graphs
* distance between two points
* midpoint of the line joining two points
* Coordinate Geometry of Straight line:
* Cartesian coordinate graph:

**Distance between two lines:**

**I**n the figure below, the coordinates of the points A and B are (x1, y1) and (x2, y2), respectively. Let the length of AB be l.

y

B(x2, y2)

l

y2 – y1

A(x1, y1) x2 – x2  C

X

Using Pythagoras theorem:

AB2 = AC2 + BC2

l2 =(x2 – x1)2 + (y2 – y1)2

**l = √(x2 – x1)2 + (y2 – y1)2**

**Example:**

Find the distance between the each pair of points: a. (3, 4) and (1, 2) b. (3, - 3) and (- 2, 5)

Solution:

Using l =√(x2 – x1)2 + (y2 – y1)2

1. l = √(3 – 1)2 + (4 – 2)2

l = √22 + 22

l = √8 = 2√2 units

1. l = √(3 – (-2)2 + (-3 – 5)2

= √52 + (-8)2

= √25 + 64 = √89 = 9.43 units

**Evaluation**: Find the distance between the points in each of the following pairs leaving your answers in surd form: 1. (-2, - 5) and (3, - 6) 2. (- 3, 4) and (- 1, 2)

**Mid-point of a line:**

The mid-point of the line joining two points:

y

B(x2, y2)

y2 - y

M(x, y) D

x2 –x

y – y1

A(x1, y1) x – x1  N C

X

Triangle MAN and BMD are congruent, so AM = MD and BD = MN

x – x1 = x2 – x y – y1 = y2 – y

x + x = x2 + x1 y + y = y2 + y1

2x = x2 + x1 2y = y2 + y1

**x= x2 + x1 y = y2 + y1**

**2 2**

Hence, the **mid-point** of a straight line joining two is **x2 + x1 ,y2 + y1**

**2 2**

**Example**: Find the coordinates of the mid-point of the line joining the following pairs of points.

1. (3, 4) and (1, 2) b. (2, 5) and ( - 3, 6)

Solution:

Mid-point = **x2 + x1 ,y2 + y1**

**2 2**

1. Mid-point = 1 + 3 , 4 + 2 = (2, 3)

2 2

1. Mid-point = - 3 + 2 , 6+ 5 = - 1 , 11

2 2 2 2

**Evaluation:** Find the coordinates of the mid-point of the line joining the following pairs of points.

1. (- 2 , - 5) and (3, - 6) b. (3, 4) and ( - 1, - 2)

**General Evaluation**

1. Find the distance between the points in each of the following pairs leaving your answers in surd form: 1. (7, 2) and (1, 6)
2. What is the value of r if the distance between the points (4, 2) and (1, r) is 3 units?
3. Find the coordinates of the mid-point (-3, -2) and (-7, - 4)

**Reading Assignment: NGM for SS 3 Chapter 9** page 77 – 78,

**Weekend Assignment:**

1. Find the value of α2 + β2 if α + β = 2 and the distance between the points (1, α) and (β, 1) is 3 units.
2. The vertices of the triangle ABC are A (7, 7), B (- 4, 3) and C (2, - 5). Calculate the length of the longest side of triangle ABC.
3. Using the information in ‘2’ above, calculate the line AM, where M is the mid-point of the side opposite A.

**WEEK THREE**

* Coordinate Geometry of straight lines:
* Gradient and Intercepts of a line
* Angle between two intersecting straight lines and application

**Gradient and Intercepts of a line**

Gradient of a line of the form y = mx + c, is the coefficient of x, which is represented by m and c is the intercept on the y axis.

**Example**

1. Find the equation of the line with gradient 4 and y-intercept -7.

**Solution**

m = 4, c = - 7,

Hence, the equation is; y =4x - 7.

**Evaluation:**

1.What is the gradient and y intercept of the line equation 3x -5y +10=0 ?

2. Find the equation of the line with gradient - 9 and y-intercept 4.

**Gradient and One Point Form**

The equation of the line can be calculated given one point (x, y) and gradient (m) by using the formula; y - y1= m(x - x1)

**Example**

Find the equation of the line with gradient -8 and point(3, 7).

**Solution**

m = - 8, (x1, y1) =(3,7)

Equation: y - 7 = - 8(x - 3)

y = -8x + 24 +7

y = -8x + 31

**Evaluation:**

1. Find the equation of the line with gradient 5 and point(-2, -7).

2. Find the equation of the line with gradient -12and point (3, -5).

**Two Point Form:**

Given two points (x1, y1) and (x2, y2), the equation can be obtained using the formula:

y2 - y1 = y - y1

x2 - x1 x - x1

Example: Find the equation of the line passing through (2,-5) and (3,6).

**Solution**

6 - (-5)/3 - 2 = y - (-5)/x - 2

11 = y + 5/x - 2

11(x - 2) = y + 5

11x - 22 = y + 5

y - 11x + 27 = 0

**Evaluation:**

1.Find the equation of the line passing through (3, 4) and (-1, -2).

2.Find the equation of the line passing through (-8, 5) and (-6, 2).

**Angles between Lines**

**Parallel lines:**

The angle between parallel lines is 00 because they have the same gradient

**Perpendicular Lines:**

Angle between two perpendicular lines is 900 and the product of their gradients is – 1. Hence, m1m2 = - 1

**Examples:**

1. Show that the lines y = -3x + 2 and y + 3x = 7 are parallel.

solution:

Equation 1: y = -3x + 2, m1 = -3

Equation 2: y + 3x = 7,

y = -3x + 7, m2 = - 3

since; m1 = m2 = - 3, then the lines are parallel

1. Given the line equations x = 3y + 5 and y + 3x = 2, show that the lines are perpendicular.

solutions:

Equation 1: x = 3y + 5, make y the subject of the equation.

3y = x + 5

y = x/3 + 5/3

m1 = 1/3

Equation 2: y + 3x = 2,

y = - 3x + 2, m2 = -3

hence: m1 x m2 = 1/3 x – 3 = - 1

since: m1m2 = - 1, then the lines are perpendicular.

**Evaluation:** State which of the following pairs of lines are: (i) perpendicular (ii) parallel

(1) y = x + 5 and y = - x + 5 (2). 2y – 6 = 5x and 3 – 5y = 2x (3) y = 2x – 1 and 2y – 4x = 8

**Angles between Intersecting Lines:**

**y**

y = mx + c

θ x

0

The gradient of y = mx + c is tan θ. Hence **m = tan θ**can be used to calculate angles between two intersecting lines. Generally the angle between two lines can be obtained using: tan 0 = m2 -m1

1 + m1m2

Example: Calculate the acute angle between the lines y=4x -7 and y = x/2 + 0.5.

Solution:

Y=4x -7, m1= 4, y=x/2+0.2, m2 =1/2.

Tan O= 0.5 - 4. = -3.5/3

1 + (0.5\*4)

Tan O =- 1.1667

O=tan-1(-1.1667) = 49.4

Evaluation:Calculate the acute angle between the lines y=3x -4 and x - 4y +8 = 0.

General Evaluation:

1.Calculate the acute angle between the lines y=2x -1 and 2y + x = 2.

2.If the lines 3y=4x -1 and qy= x + 3 are parallel to each other, find the value of q.

3.Find the equation of the line passing through (2,-1) and gradient 3.

**Reading Assignment: NGM for SS 3 Chapter 9** page 75 – 81

Weekend Assignment

1.Find the equation of the line passing through (5,0) and gradient 3.

2.Find the equation of the line passing through (2,-1) and (1, -2).

3. Two lines y=3x - 4 and x - 4y + 8=0 are drawn on the same axes.

(a) Find the gradients and intercepts on the axes of each line.

(b) Find the equation parralel to x -4y + 8=0 at the point (3, -5)

**WEEK FOUR**

* Differentiation of algebraic functions: meaning of differentiation
* Differentiation from first principle
* Standard derivatives of some basic functions.

Consider the curve whose equation is given by y = f(x) Recall that m = y2 – y1= f(x+x)-f(x)

x2- x1x

As point B moves close to A, dx becomes smaller and tends to zero.

The limiting value is written on Lim f(x +x) – f(x) and is denoted by as x –> 0

dx

fl(x) is called the **derivative of f(x)** and the **gradient function of the curve**

The process of finding the derivative of f(x) is called differentiation. The rotations which are commonly used for the derivative of a function are f1(x) read as f – prime of x, df/dx read as dee x of f

df/dx read dee - f dee- x, dy/dx read dee - y dee- x

**If y = f(x) , this dy/dx = f1(x) (it is called the differential coefficient of y with respect to x.**

**Differentiation from first principle:** The process of finding the derivative of a function from the consideration of the limiting value is called differentiation from first principle.

**Example 1**

Find from first principle, the derivative of y = x2

Solution

y = x2

y + y = (x + x)2

y + y = x2 + 2xx + (x)2

y = x2 + 2xx+ (x)2  - y

y = x2 + 2xx + (x)2 - x2

y = 2xx + (x)2

y = (2x + x)x

y = 2x + x

x

Lim x = 0

dy = 2x

dx

**Example 2**:

Find from first principle, the derivative of 1/x

Solution

Let y = 1

x

y + y = 1

x + x

y = 1 - y

x + x

y = 1 - 1

x + x x

y = x – (x + x)

(x +x)x

y = x - x - x

x2 + xx

dy = -x

x2+ x

y = -1

x x2 + x

Lim x = 0

dy = -1

dx x2

**Evaluation:** Find from first principle, the derivatives of y with respect to x:

1. Y = 3x3 2. Y = 7x2 3. Y = 3x2 – 5x

**Rules of Differentiation:** Let y = xn

y + dy = (x + dx)n

= xn + nxn-1dx + n(n -1) xn-2(dx)2 + … (dx)n

2!

= xn + n xn-1dx + n(n-1) xn-2 (dx)2+ --- + (dx)n - xn

2!

= nxn-1dx + n (n – 1) xn–1 (dx)2

2!

dy/dx = n xn-1 + n (n –1) xn-1 dx

Lim dy/dx = nxn-1

dx = 0

Hence; **dy/dx = nxn-1 if y = xn**

**Example 3**:

Find the derivative of the following with respect to x: (a) x7 (b) x½ (c) 5x2 – 3x (d) - 3x2 (e) y = 2x3 – 3x + 8

Solution

a. Let y = x7

dy/dx = 7 x7-1 = 7x6

b. Let y = x ½

dy/dx = ½ x½ -1 = ½ x– ½ = 1

2√x

c. Let y = 5x2 – 3x

dy/dx = 10x – 3

d. Let y = - 3x2

dy/dx =2× - 3x2-1 = - 6x

e. Let y = 2x3 – 3x + 8

dy/dx= 3 x 2x3-1 – 3 + 0

= 6x2 – 3

**Evaluation:**

1. If y=5x4 ,find dy/dx 2.Given that y= 4x-1 find y1

**General Evaluation**

1. Find, from first principles, the derivative of 4x2 – 2 with respect to x.

2. Find the derivative of the following a.3x3 – 7x2 – 9x + 4 b. 2x3 c. 3/x

**3.** Using idea of difference of two square; simplify 243x2 - 48y2

4. Expand (2x -5)( 3x-4)

5. If the gradient of y=2x2-5 is -12 find the value of y.

**Reading Assignment: NGM for SS 3 Chapter 10** page 82 -88,

**Weekend Assignment**

**Objective**

1. Find the derivative of 5x3(a) 10x2 (b) 15x2 (c) 10x (d) 15x3

2. Find dy/dx, if y = 1/x3(a) –3/x4 (b) 3/x4  (c) 4/x3 (e) –4/x3

3. Find f1(x), if f(x) = x3 (a) 3x (b) 3x2 (c) ½ x3 (d) 2x3

4. Find the derivative of 1/x(a) 1/x2 (b) –1/x2 (c) – x (d) –x2

5. If y = - 2/3 x3. Find dy/dx (a) 4/3 x2  (b) 2x2 (c) – 2x2 (d) –2x

**Theory**

1. Find from first principle, the derivative of y = x + 1/x

2. Find the derivative of 2x2 – 2/x3

**WEEK FIVE**

* Differentiation of algebraic functions:
* Basic rules of differentiation such as sum and difference, product rule, quotient rule
* Maximal and minimum application.

**Derivative of algebraic functions**

Let f, u, v be functions such that

f(x) = u(x) + v(x)

f(x +x ) = u(x +x ) + v(x + x)

f(x + x) – f(x) = {u(x+ x) + v(x+ x) – v(x + x) – u(x) – v(x)}

= u (x +x ) – u(x) + v(x +x ) – v(x)

f(x + x) – f(x) = u(x +x)-u(x) + v(x +x ) – v(x)

Lim f(x + x) – f(x) = U1(x) + V1(x)

if y = u + v and u and v are functions of x, then **dy/dx = du/dx + dv/dx**

**Examples**:Find the derivative of the following

1) 2x3 – 5 x2 + 2 2)3x2 + 1/x 3)2x3 + 2x2 +1

Solution

1. Let y = 2x3 – 5x2 + 2

dy/dx = 6x2 – 10x

2. Let y = 3x2+ 1/x = 3x2 + x-1

dy/dx = 6x – x-2 = 6x – 1

x2

3. Let y = 2x3 + 2x2 + 1

dy/dx=6x2 + 4x

**Evaluation:** 1. If y = 3x4 – 2x3 – 7x + 5. Find dy/dx

2.Findd (8x3 – 5x2 + 6)

Dx

**Function of a function (chain rule)**

Suppose that we know that y is a function of u and that u is a function of x, how do we find the derivative of y with respect to x?

Given that y = f(x) and u = h(x), what is dy/dx?

dy/dx = , this is called the chain rule

**Examples**

Find the derivative of the following.(a)y = (3x2 – 2)3 (b) y = (1 – 2x3) (c) 5/(6-x2)3

Solution

1. y = (3x2 – 2)3

Let u = 3x2 – 2

y = (3x2 – 2)3 => y = u3

y = u3

dy/du = 3u2

du/dx = 6x

dy/dx = = 3u2 x 6x

= 18xu2 = 18x(3x2 – 2)2

2. y = (1 – 2x3)1/2 => (1 – 2x3) 1/2

Let u =1 – 2x3, hence y = u1/2

dy/dx = = ½ u-1/2 x( –6x2)

= -3x2 u – ½= -3x2

u1/2

-3x2 = -3x2

√ u √(1 – 2x3)

3. y = 5 = 5(6 – x2)-3

(6 – x2)3

Let u = 6 – x2

y = 5(u)–3

dy/du = -15u –4

du/dx = -2x

dy/dx = dy/du X du/dx = -15u-4 x (-2x) = 30x u-4 = 30x (6 – x2)-4

= 30x\_

(6 – x2)4

**Evaluation**:

1. Given that y = 1 find dy/dx

(2x + 3)4

2. If y = (3x2 + 1)3 , Find dy/dx

**Product Rule**

We shall consider the derivative of y = uv where u and v are function of x.

Let y = uv

Then y + y = (u +u )(v + v)

= uv + uv + vu +uv

y = uv + uv + vu+ uv – uv

y= uv + vu + uv

y= uv + vu + uv

x x

As x =>0 ,u=> 0 , v=> 0

Lim y = Lim uv + Lim vu + Lim uv

x=>0 x x=>0 x x=>0 x x=>0 x

Hence **dy/dx**= **U dv + Vdu**

**dxdx**

**Examples**

Find the derivatives of the following.

(a) y = (3 + 2x) (1 – x) (b) y = (1 – 2x + 3x2) (4 – 5x2)

Solution

1. y = (3 + 2x) (1 – x)

Let u = 3 + 2x and v = (1 – x)

du/dx = 2 and dv/dx = -1

dv/dx = u dv + vdu

dx dx

= (1 – x) 2 + (3 + 2x) (-1) = 2 – 2x – 3 – 2x

dy/dx = - 1 – 4x

2. y = (1 – 2x + 3x2) (4 – 5x2)

Let u = (1 – 2x + 3x2) and v = (4 – 5x2)

du/dx = -2 + 6x and dv/dx = - 10x

dy/dx = udv + vdu

dxdx

= (1 – 2x + 3x2) (-10x) + (4 – 5x2) (- 2 + 6x)

= - 10x + 20x2 – 30x3 + (- 8 + 10x2 + 24x – 30x3)

= - 10x + 20x2 – 30x3 – 8 + 10x2 + 24x – 30x3

= 14x + 30x2 – 60x3 – 8

**Evaluation**

Given that (i) y = (5+3x)(2-x) (ii) y = (1+x)(x+2)3/2 **,**find dy/dx

**Quotient Rule:**

If y = u

**v**

then; **dy = vdu- udv**

**dxdxdx**

**v2**

**E**xamples:

Differentiate the following with respect to x. (a)x2 + 1 (b)(x – 1)2

1 – x2 √x

Solution:

1. y = x2 + 1

1 – x2

Let u = x2 + 1 du/dx = 2x

v= 1 – x2  dv/dx = - 2x

dy = vdu- udv

dxdxdx

**v2**

dy/dx = (1 – x2)(2x) – (x2 + 1)(-2x)

(1 – x2)2

= 2x – 2x3 + 2x3 + 2x

(1 – x2)2

**dy/dx = 4x**

1. – x2)2
2. y = (x – 1)2

√x

Let u = (x – 1)2 du/dx = 2(x – 1)

v = √x dv/dx = 1/2√x

dy/dx = √x 2(x - 1) -(x – 1)2 1/2√x

(√x)2

dy/dx = √x 2(x - 1) - (x – 1)2 1/2√x

x

**Evaluation:** Differentiate with respect to x: (1) (2x + 3)3  (2) √x

(x3– 4)2 √(x + 1)

**Applications of differentiation:**

There are many applications of differential calculus.

Examples:

1. Find the gradient of the curve y = x3 – 5x2 + 6x – 3 at the point where x = 3.

Solution:

Y = x3 – 5x2 + 6x – 3

dy/dx = 3x2 – 10x + 6

where x = 3; dy/dx = 3(32) – 10(3) + 6

= 27 – 30 + 6

= 3.

1. Find the coordinates of the point on the graph of y = 5x2 + 8x – 1 at which the gradient is – 2

Solution:

Y = 5x2 + 8x – 1

dy/dx = 10x + 8

replace; dy/dx by – 2

10x + 8 = - 2

10x = - 2 – 8

x = -10/10 = - 1

1. Find the point at which the tangent to the curve y = x2  - 4x + 1 at the point (2, -3)

Solution:

Y =x2  - 4x + 1

dy/dx = 2x – 4

at point (2, -3): dy/dx = 2(2) – 4

dy/dx = 0

tangent to the curve: y – y1 = dy/dx(x – x1)

y – (-3) = 0 ( x- 2)

y + 3 = 0

**Evaluation**:

1. Find the coordinates of the point on the graph of y = x2 + 2x – 10 at which the gradient is 8.

2. Find the point on the curve y = x3 + 3x2 – 9x + 3 at which the gradient is 15.

**Velocity and Acceleration**

**Velocity**: The velocity after t seconds is the rate of change of displacement with respect to time.

Suppose; s = distance and t = time,

Then; ***Velocity = ds/dt***

**Acceleration**: This is the rate of change of velocity compared with time.

***Acceleration = dv/dt***

Example:

A moving body goes s metres in t seconds, where s = 4t2 – 3t + 5. Find its velocity after 4 seconds. Show that the acceleration is constant and find its value.

Solution:

S = 4t2 – 3t + 5

ds/dt = 8t – 3

velocity = ds/dt = 8(4) – 3

= 32 – 3

= 29

Acceleration: dv/dt = 8.

**Maxima and Minimal**

1. Find the maximum and minimum value of y on the curve 6x – x2.

Solution:

y = 6x – x2

dy/dx = 6 – 2x

equatedy/dx = 0

6 – 2x = 0

6 = 2x

X = 3

The turning point is (3, 9)

1. Find the maximum and minimum of the function x3 – 12x + 2.

Solution:

Y =x3 – 12x + 2

dy/dx = 3x2 – 12

3x2 – 12 = 0

3x2 = 12

x2 = 12/3

x2 = 4 x = ± 2

minimum point occur when d2y/dx2> 0

maximum point occurs when d2y/dx2< 0

d2y/dx2= 6x

substitute x = 2; d2y/dx2 = 6 x 2 = 12

therefore: the function is minimum at point x = 2 and y = - 14

substitute x = - 2; d2y/dx2 = 6(-2) = -12

therefore: the function is maximum at point x = - 2 and y = 18

**Evaluation:**

1. A particle moves in such a way that after t seconds it has gone s metres, where s = 5t + 15t2 – t3

2. Find the maximum and minimum value of y on the curve 4 –12x - 3x2.

**General Evaluation**

Use product rule to find the derivative of

1. y = x2 (1 + x)½

2. y = √x (x2 + 3x – 2)2

3. Find the derivative of y =(7x2 -5)3

4. Using completing the square method find t if s=ut+1at2

2

5. If 3 is a root of the equation x2 – kx +42=0 find the value of k and the other root of the equation

**READING ASSIGNMENT: NGM for SS 3 Chapter 10** page 90 -101,

**WEEKEND ASSIGNMENT**

**OBJECTIVE**

1.Differentiate the function 4x4 + x3 – 5 (a)4x3 +3x2 (b)16x2 +3x2 (c)16x3 +3x2 (d)16x4 + 3x2

2.Find d2y/dx2 of the function y = 3x5wrt x. (a) 15x3 (b) 45 x4 (c) 60x3 (d) 3x5 (e) 12x3

3.If f(x) = 3x2 + 2/x find f1(x) (a) 6x + 2 (b) 6x + 2/x2 (c) 6x – 2/x2 (d)6x -2

4.Find the derivative of 2x3 – 6x2 (a) 6x2 – 12x (b) 6x2 – 12x (c) 2x2 – 6x (d) 8x2 – 3x

5.Find the derivative of x3 – 7x2 + 15x (a) x2 – 7x + 15 (b) 3x2 – 14x + 15 (c) 3x2 + 7x + 15 (d) 3x2 – 7x + 15

**THEORY**

1. Differentiate with respect to x. y2 + x2 – 3xy = 4

2. Find the derivative of 3x3(x2 + 4)2